A study of Morphological Computation by using Probabilistic Inference for Motor Planning

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***Abstract*—One key idea behind morphological computation is that many difficulties of a control problem can be absorbed by the morphology of the robot. The performance of the controlled system naturally depends on the control architecture and on the morphology of the robot. Ideally, adapting the morphology of the plant and optimizing the control law interact such that finally, optimal physical properties of the system and optimal control laws emerge. As a first step towards this vision we propose to use optimal control methods for investigating the power of morphological computation. We use probabilistic inference for motor control to acquire optimal control laws given the current morphology. By changing the morphology of our robot, control problems can be simplified, resulting in controllers with higher performance and reduced complexity.**

***Keywords*—probabilistic motor planning, stochastic optimal control, morphological computation**

I. Introduction

The term ’Morphological Computation’ denotes the computation done by the morphology of a plant in contrast to the computation done by the controller of the plant. Despite of the advancements for theoretical models of morphological computation [1], [2] it is often hard to quantify how much computation is done by the morphology of a real robot and how much by the controller. For a given morphology, the performance of the robot-controller loop heavily depends on the used control architecture. Therefore, we will use optimal control methods to eliminate this dependency. For each morphology, we will use its optimal control law.

As optimal control method, we will use Approximate Inference Control (AICO) [3], which is a state of the art planning method for stochastic optimal control tasks. AICO is based on probabilistic inference for motor control. The beauty of this approach is that there is no distinction between sensor and motor, perception and action. We can include a multitude of variables, some of which might represent some features of the state, some of which might represent goals, constraints or motivations in the future and some of which might represent future actions or motor signals.

As all other stochastic optimal control methods, AICO minimizes a cost function, which is in our case given by the quadratic distance to a target state and the used energy of the movement. In order to apply the AICO method to torque constraint dynamical models we will briefly explain how to extend the algorithm to systems with control limits.

AICO also provides us with a time-varying linear feedback controller as policy. We will use the variance of the control gains as complexity of the controller. If the control gains are almost constant in time, the control law is close to linear and does not need much computation. However, if the control gains are highly varying, the controller needs to do a lot of computation and therefore, less computation is provided by the morphology in order to fulfill a task.

We will illustrate the power of morphological computation combined with optimal control on a 2-link robot which has to keep balance in the presence of external pushes. We will show that by changing the morphology of a robot like the joint friction, the resulting optimal controllers have reduced complexity. As we will show there is an optimal value for this physical property for a given control task.

II. Probabilistic Inference for Motor Planning

Stochastic Optimal Control (SOC) provides attractive methods to build non-linear, noise robust optimal control laws for high dimensional robot control tasks [4], [5], [3]. We will use the Approximate Inference Control (AICO) [3] algorithm, which is a SOC method based on probabilistic inference for motor control. Most applications of AICO are in the kinematic planning domain. Here, we want to apply AICO to fully dynamic, torque controlled robot simulations. Therefore we had to extend the AICO framework with control or torque limits, which will be explained in the next sections.

A. Approximate Inference Control

We will briefly clarify the notation for our discussion. Let $\mathbf{q}$ denote the state and $\mathbf{u}$ the control vector. A trajectory τ is defined as sequence of state control pairs, $\tau = \langle \mathbf{q}_{0:T}, \mathbf{u}_{0:T-1} \rangle$, where T is the length of the trajectory. Each trajectory has associated costs

$$L(\tau) := \sum_{t=0}^{T} c_t(\mathbf{q}_t, \mathbf{u}_t). \quad (1)$$

AICO uses message passing in graphical models to infer the optimal trajectory τ. In order to transform the minimization of $L(\tau)$ into an inference problem, for each time step an individual binary random variable z_t is introduced. This random variable indicates a reward event. Its probability is given by $P(z_t = 1|\mathbf{q}_t, \mathbf{u}_t, t) \propto \exp(-c_t(\mathbf{q}_t, \mathbf{u}_t))$, where $c_t(\mathbf{q}_t, \mathbf{u}_t)$ represents the cost function for a single time step. AICO now assumes that a reward event $z_t = 1$ is observed at every time step. Given that evidence, AICO calculates the posterior distribution $P(\mathbf{x}_{1:T}, \mathbf{u}_{1:T}|z_{1:T} = 1)$ over trajectories.

We will use the simplest version of AICO, where an extended Kalman smoothing approach is used to estimate the posterior. Therefore, the non-linear system is approximated by a Linear dynamics, Quadratic costs and Gaussian noise system (LQG) by Taylor expansions. For this LQG system a simple message passing algorithm can be used, which uses only Gaussian messages. The messages are used to calculate a new belief over the trajectories. This belief is again used to calculate a LQG system by Taylor expansions. For algorithmic details we refer to [3].

If we use AICO with a constant cost and dynamic model for each time step, the algorithm reduces to calculating a Linear Quadratic Regulator (LQR), which is often used in optimal control. An LQR is the optimal linear feedback controller for a given linear system. In contrast, AICO uses time-varying linear feedback controllers, the feedback controller may be different for each time step. We will also evaluate the benefit of using AICO (time-varying linear feedback control) and an LQR (constant linear feedback control) in our experiments.

B. Cost Function for Constrained Systems

In order to apply the AICO algorithm to torque controlled robots, we have to extend the framework to incorporate control limits as the available motor torque is typically limited. This is done by adding a control dependent punishment term to the cost function if the current mode of the belief of the controls is outside the control limits.

For example, if the current mode of the belief of $\mathbf{u}_t$ exceeds a given bound $\mathbf{u}_{B_t}$ at time t, we add the following term to our cost function c_t:

$$c_t^{\text{lim}}(\mathbf{u}_t) = (\mathbf{u}_t - \mathbf{u}_B)^T \mathbf{H}_{B_t} (\mathbf{u}_t - \mathbf{u}_B).$$

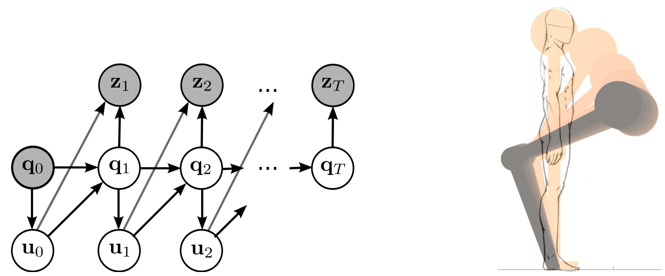
As a consequence, the resulting Gaussian distributions which are used to represent the costs c_t change. This distributions have typically zero mean in the control space due to the typically used quadratic control costs. In the case of control limits the mean of the distribution is *non-zero*. Consequently, also the message passing update equations used for the AICO algorithm changes. For the exact message passing equations of AICO with control limits we refer to [6].

III. Experiments

We investigate morphological computation combined with optimal control on a dynamic non-linear balancing task [7] where a robot gets pushed from behind with a specific force and has to move such that it maintains balance. The optimal strategy is a non-linear control law which returns the robot to the upright position, see Figure 1. We quantify morphological computation by changing the joint friction.

A. Setting

In our experiments we model a humanoid with a 2-link pendulum. In the initial setting, both links have a length of 1m



(a) Graphical model for planning (b) Balancing task

Fig. 1. (a) Graphical Model for Probabilistic Planning. The state variable $\mathbf{q}_t$ denotes the joint angles and joint velocities. Controls are labelled by $\mathbf{u}_t$. The time horizon is fixed to T time steps. The task variable z_t expresses a performance criteria (like reaching a goal). (b) The agent modelling a human (2m, 70kg) gets pushed from behind and has to move such that it keeps balance.

and a mass of 35kg. The ankle and the hip angle are actuated, where the torques $\mathbf{u}$ lie in the interval $[\pm 70, \pm 500]$Ns. The ankle angle is limited to the range $[-0.4, 0.8]$ and the hip angle to the range $[-1.6, 0.1]$. The trajectory was simulated with a time-step of 0.5ms, for the AICO algorithm we used a time step of $\Delta t = 10$ms and $T = 2$s, which resulted in a planning horizon of 200 time steps.

We use a quadratic cost function given by

$$c_t(\mathbf{q}_t, \mathbf{u}_t) = (\mathbf{q}_t - \mathbf{q}_T)^T \hat{\mathbf{R}}_t (\mathbf{q}_t - \mathbf{q}_T) + \mathbf{u}_t^T \hat{\mathbf{H}}_t \mathbf{u}_t,$$

where $\hat{\mathbf{H}}_t = 10^{-5}\mathbf{I}$ is the control precision matrix and $\hat{\mathbf{R}}_t = \text{diag}(2000, 20, 2000, 20)$ is the state precision matrix.

We have multiplied the controls by Gaussian zero mean noise denoted by ϵ with a variance of 0.1, which corresponds to 10% torque dependent noise. This noise affects the system dynamics $\dot{\mathbf{x}} = f(\mathbf{x}_t, \mathbf{u}_t + \epsilon)$ while simulating a trajectory, where $\epsilon = \mathcal{N}(0|0.1 \cdot \text{abs}(\mathbf{u}_t))$.

B. Complexity Measures

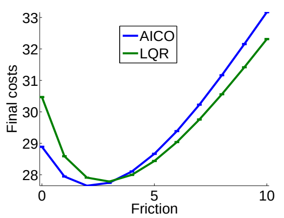
The experiments are performed for multiple forces $F = [2.5, 5, 7.5]$Ns and multiple initial states with the hip angles $\phi_2 = [0, -4, -7, -10]\pi/180$. We evaluate the average values of the final costs $L(\tau)$ (see Equation 1) and the total jerk $J(\tau) = \Delta t \sum_{t=0}^{T} \dot{\mathbf{u}}_t^T \dot{\mathbf{u}}_t$ of the trajectory (proportional to the squared derivative of the torque). Different complexity measures would be possible, the chosen ones are plausible since the complexity of controlling the pendulum is reflected by how well the costs $L(\tau)$ are optimized and by the complexity of control signal. As the jerk of a movement tracks the derivative of the torques, the jerk seems to be a reliable complexity measure.

In addition we also calculate the variance of the time-varying controller gains returned by the AICO approach. This measure quantifies the complexity of the control law in comparison to linear controllers (no variance).

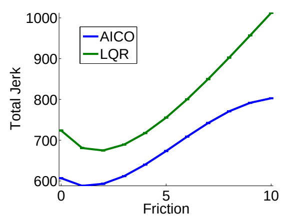
C. Friction induces Morphological Computation

In this experiment we evaluate the influence of the friction coefficient γ on the quality of the optimal controller. The friction coefficient directly modulates the acceleration of the joints, i.e. $\ddot{\phi}_\gamma = -\gamma\dot{\phi}$.

Figure 2(a) shows the resulting costs for different friction coefficients. The most simple control law according to the jerk criteria can be found at $\gamma = 2$ in Figure 2(b). We also evaluated the performance of the LQR controller, which is a constant linear feedback controller in contrast to the controller obtained by AICO, which uses a time-varying linear feedback controller. The AICO controllers showed reduced costs as well as a reduced jerk of the trajectory. In Figure 3, we can see the variance of the time-varying controller gains, which we use as complexity measure of the controllers. As we can see, the complexity of the resulting controller could also be reduced by changing the friction coefficient. The minimum of the complexity lies in the same range as the minimum of the cost function. Thus, in this case a simpler controller resulted in better performance because the morphology (the friction) of the robot has contributed to the control task.

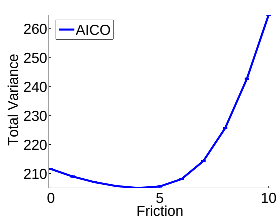


(a) Friction vs. Final Costs



(b) Friction vs. Jerk

Fig. 2. Influence of the friction coefficient on costs and jerk of the trajectory.



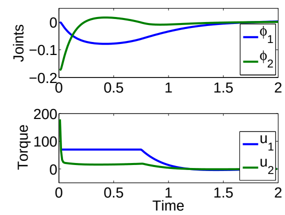
(a) Friction vs Controller Variance

Fig. 3. Influence of the friction coefficient on the complexity of the planning task. We characterize the complexity of the planning task by the variance of the time-varying feedback controller gains which are returned by AICO.

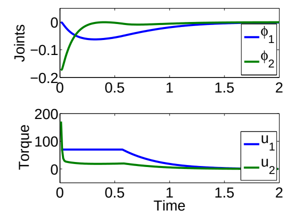
In Figure 4 we can see 3 joint and torque trajectories for different friction coefficients.

IV. Conclusion and Future Work

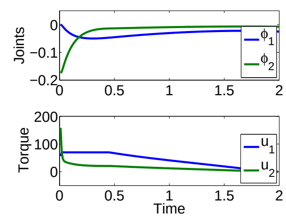
In this paper we have shown that optimal control and morphological computation are two complementary approaches which can benefit from each other. The search for an optimal morphology is simplified if we can calculate an optimal controller for a given morphology. This calculation can be done by new approaches from probabilistic inference for motor control, i.e. the AICO algorithm. By the use of optimal control methods, we have shown that for a simple, but non-linear balancing task, an appropriate setting of the friction can simplify the control task.



(a) $\gamma = 0$



(b) $\gamma = 3$



(c) $\gamma = 10$

Fig. 4. The plot shows the trajectories using the AICO approach for the friction coefficients $\gamma = 0$, $\gamma = 3$ and $\gamma = 10$.

In the future, we plan to investigate more complex and more non-linear tasks. In this case the benefit of AICO in comparison to LQR controllers should be even more prominent. In the end we are planning to simultaneously evolve walking controllers based on AICO and the morphology of bipedal robots like a model of a planar walker.

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References

[1] H. Hauser, R. Pfeifer, A. J. Ijspeert, and W. Maass, “A theoretical foundation for morphological computation,” 2011, submitted for publication.

[2] R. Pfeifer, M. Lungarella, and F. Iida, “Self-organization, embodiment, and biologically inspired robotics,” *Science*, vol. 318, no. 5853, pp. 1088–1093, 2007.

[3] M. Toussaint, “Robot trajectory optimization using approximate inference,” in *Proceedings of the 26th Annual International Conference on Machine Learning*, ser. (ICML 2009), Montreal, Canada, 2009, pp. 1049–1056.

[4] E. Todorov and W. Li, “A generalized iterative lqg method for locally-optimal feedback control of constrained nonlinear stochastic systems,” in *Proceedings of the 24th American Control Conference*, ser. (ACC 2005), vol. 1, Portland, Oregon, USA, 2005, pp. 300 – 306.

[5] H. J. Kappen, “An introduction to stochastic control theory, path integrals and reinforcement learning,” in *Cooperative Behavior in Neural Systems*, ser. American Institute of Physics Conference Series, J. Marro, P. L. Garrido, & J. J. Torres, Ed., vol. 887, Feb. 2007, pp. 149–181.

[6] E. Rückert and G. Neumann, “Approximate inference control with torque constraints,” Institute for Theoretical Computer Science, Graz University of Technology, Tech. Rep., 2011. [Online]. Available: http://www.igi.tugraz.at/rueckert/Reports/aicocs.pdf

[7] C. Atkeson and B. Stephens, “Multiple balance strategies from one optimization criterion,” in *Humanoid Robots, 2007 7th IEEE-RAS International Conference on*, Pittsburgh, USA, 2007, pp. 57 –64.